

Linear eqⁿ: A first order D.E. is linear if it is of the form $\frac{dy}{dx} + P(x)y = Q(x)$ where P & Q are fⁿ of x only or constant - ①

another form: $\frac{dx}{dy} + P(y)x = Q(y)$ ②

Working Rule: 1) I.F. for ① = $e^{\int P(x) dx}$

2) sol: $y \cdot \text{I.F.} = \int Q(x) \text{I.F.} + C$

or
1) I.F. for ② = $e^{\int P(y) dy}$

2) sol: $x \cdot \text{I.F.} = \int Q(y) \text{I.F.} + C$

M.W. $\cos y \sin 2x dx + (\cos^2 y - \cos^2 x) dy = 0$ I.F. = $\frac{1 + \sin^2 y}{\cos y}$

Q. 1. $\frac{dy}{dx} + \left(2 + \frac{1}{x}\right)y = e^{-2x}$

$P(x) = 2 + \frac{1}{x}$ $Q(x) = e^{-2x}$

I.F. = $e^{\int 2 + \frac{1}{x} dx} = e^{2x + \log x} = e^{2x} \cdot e^{\log x} = x e^{2x}$

Sol: $y \cdot x e^{2x} = \int e^{-2x} \cdot x e^{2x} dx + C$

$y \cdot x e^{2x} = \frac{x^2}{2} + C$

$y = \frac{x}{2} e^{-2x} + \frac{C}{x} e^{-2x}$ Ans.

Q. 2. $dx + x dy = e^{-y} \sec^2 y dy$

$\frac{dx}{dy} + x = e^{-y} \sec^2 y$

$P(y) = 1$

$Q(y) = e^{-y} \sec^2 y$

$$P(y) > 1 \quad \alpha \dots$$

$$I.F. = e^{\int P(y) dy} = e^{\int 1 dy} = e^y$$

$$\text{Sol: } x \cdot e^y = \int \cancel{e^y} \sec^2 y \cdot e^y dy + C$$

$$x \cdot e^y = \tan y + C$$

$$x = e^{-y} (\tan y + C) \quad \underline{\underline{\text{Ans.}}}$$

Eqnⁿ - reducible to Linear D.E :

$$\text{Bernoulli's eqn: } \frac{dy}{dx} + P(x)y = Q(x)y^n \quad n \neq 0 \text{ or } 1.$$

$$y^{-n} \frac{dy}{dx} + P(x)y^{-n+1} = Q(x) \quad \text{--- (3)}$$

$$\text{Let } y^{-n+1} = v$$

$$(1-n)y^{-n} \frac{dy}{dx} = \frac{dv}{dx} \quad \left| \quad y^{-n} \frac{dy}{dx} = \frac{1}{1-n} \frac{dv}{dx} \right.$$

$$\text{(3)} \Rightarrow \frac{1}{1-n} \frac{dv}{dx} + P(x)v = Q(x)$$

$$\frac{dv}{dx} + (1-n)P(x)v = (1-n)Q(x)$$

Linear in v.

$$I.F. = e^{\int (1-n)P(x) dx}$$

$$\text{Sol: } v \cdot (I.F.) = \int (1-n)Q(x) \cdot (I.F.) + C$$

$$\underline{\text{Q. 3}} \quad 2 \frac{dy}{dx} = \frac{y}{x} + \frac{y^2}{x^2}$$

$$2 \frac{dy}{dx} - \frac{y}{x} = \frac{y^2}{x^2}$$

$$2y^{-2} \frac{dy}{dx} - \frac{1}{x} \cdot \frac{1}{y} = \frac{1}{x^2}$$

$$P(x) = -\frac{1}{x}$$

$$Q(x) = \frac{1}{x^2}$$

$$\frac{1}{y} = v$$

$$-\frac{1}{y^2} dy = \frac{dv}{dx}$$

$$-2 \frac{dv}{dx} - \frac{1}{x} v = \frac{1}{x^2}$$

$$y^2 \frac{dy}{dx}$$

$$\frac{dv}{dx} + \frac{1}{2x} v = -\frac{1}{2x^2}$$

$$\text{I.F.} = e^{\int \frac{1}{2x} dx} = e^{\frac{1}{2} \int \frac{1}{x} dx} = e^{\frac{1}{2} \log x} = e^{\log \sqrt{x}} = \sqrt{x}$$

$$\text{Sol: } v \cdot \sqrt{x} = \int -\frac{1}{2x^2} \sqrt{x} dx + C$$

$$= -\frac{1}{2} \int \frac{1}{x^2} \cdot \sqrt{x} dx + C$$

$$= -\frac{1}{2} \int x^{-3/2} dx + C$$

$$= -\frac{1}{2} \frac{x^{-3/2+1}}{-3/2+1} + C$$

$$= -\frac{1}{2} \cdot \frac{-2}{1} x^{-1/2} + C$$

$$v \cdot \sqrt{x} = 2 + \frac{1}{\sqrt{x}} + C$$

$$\frac{1}{y} = 2 + \frac{1}{x} + \frac{C}{\sqrt{x}}$$

Ans.

$$\frac{1}{x^2} \cdot x^{1/2} = \frac{1}{x^{2-1/2}} = \frac{1}{x^{3/2}}$$